3D Scene modelling from multiple range views

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ABSTRACT

This paper presents a new 3D scene analysis system that automatically reconstructs the 3D geometric model of real-world scenes from multiple range images acquired by a Laser Range Finder on board of a mobile robot. The reconstruction is achieved through an integrated procedure including range data acquisition, geometrical feature extraction, registration and integration of multiple views. Different descriptions of the final 3D scene model are obtained: a polygonal triangular mesh, a surface description in terms of planar and biquadratics surfaces and a 3D boundary representation. Relevant experimental results from the complete 3D scene modelling are presented. Direct applications of this technique include 3D reconstruction and/or update of architectural or industrial plans into a CAD model, design verification of buildings, navigation of autonomous robots, and input to virtual reality systems.

Keywords: Range images, multiple views, 3D scene modelling, hybrid segmentation, shape and object representation, CAD-based Vision.

1. INTRODUCTION

There has been much research into the use of range scanners (either laser time-of-flight or triangulation) for scanning environments for robot path planning, or for providing data for object recognition or part inspection. Simultaneously, there has been industrial interest into the creation of CAD models from already existing industrial parts. However, there has not been much research in the combination of these themes for the automatic construction of pre-existing environments, i.e., modelling from reality. Modelling from observation seeks to generate a CAD model of an object from multiple range/intensity images by using a combination of computer vision and computer graphics techniques.

This paper presents a new 3D scene analysis system that automatically reconstructs the 3D geometric model of real-world complex scenes (distance range from 0.5 to 20 meters) from multiple range images acquired by a Laser Range Finder on board of a mobile robot. The reconstruction is achieved through an integrated procedure including range data acquisition, geometrical feature extraction, registration and integration of multiple views. Multiple range images from different viewpoints are acquired to resolve the ambiguities caused by occlusions and to construct a complete 3D model. Occlusions are detected based on surface discontinuities (jump edges) and used in the perception planning process to reason about the next viewpoint. Fig. 1 is an overview of the iterative procedure for 3D reconstruction.

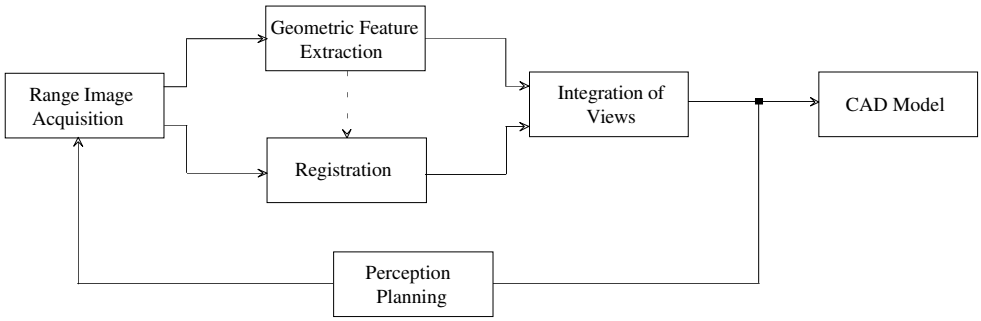


Fig. 1. Iterative procedure for 3D reconstruction.

The paper's novelty is the combination of different algorithms to overcome the specific problems raised by this kind of images. Namely, a new hybrid algorithm for range image segmentation is described. This algorithm combines edge and region detection approaches to build a high-level description of a scene given a noisy range image. Another relevant issue is the consistent registration/integration of range images from different viewpoints. Emphasis is given to the registration of multiple partially overlapping range images. A hierarchical registration scheme working at the pixel level for higher accuracy is described. The integration of different views works, instead, at a higher representation level, e.g., edges and surfaces. Different descriptions of the final 3D scene model are obtained: a polygonal triangular mesh, a surface description in terms of planar and biquadratics surfaces and a 3D boundary representation.

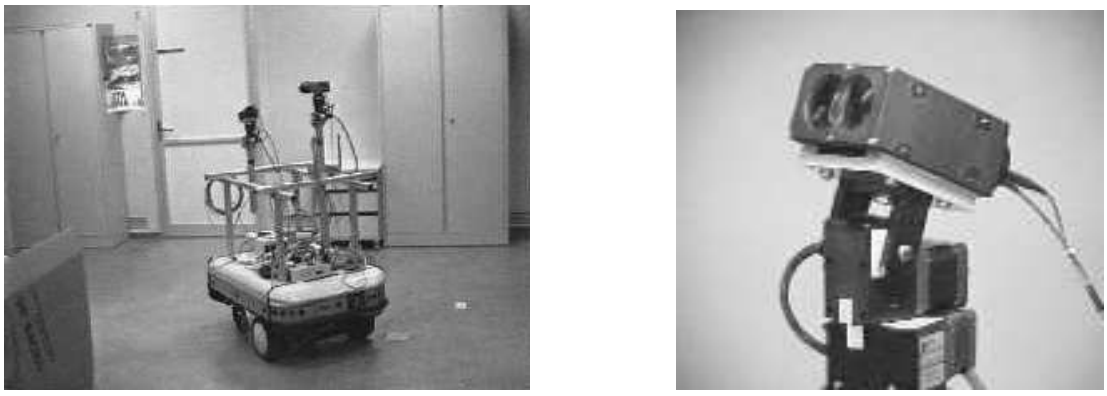
Relevant experimental results from the complete 3D scene modelling are presented. Direct applications of this technique include 3D reconstruction and/or update of architectural or industrial plans into a CAD model, design verification of buildings, navigation of autonomous robots, and input to virtual reality systems.

In Section 2 the current procedure for range image acquisition is described. This is followed by Section 3 on 3D surface reconstruction from a single view, describing the geometric feature extraction. Section 4 concentrates on the combination of multiple range images for 3D scene modelling. Two different techniques are described yielding different high-level descriptions of the scene: a polygonal triangular mesh, a surface description in terms of planar and bi-quadratic surfaces and respective 3D boundary representation. Experimental results from a complex environment are presented in Section 5. Finally, conclusions and work in progress are outlined in Section 6.

2. DISTANCE MEASUREMENTS

A large variety of measurement techniques can be used to create a range image, including structured light, time-of-flight lasers, radar, sonar, and vision based methods namely depth from stereo, shading, texture, motion and focus [1].

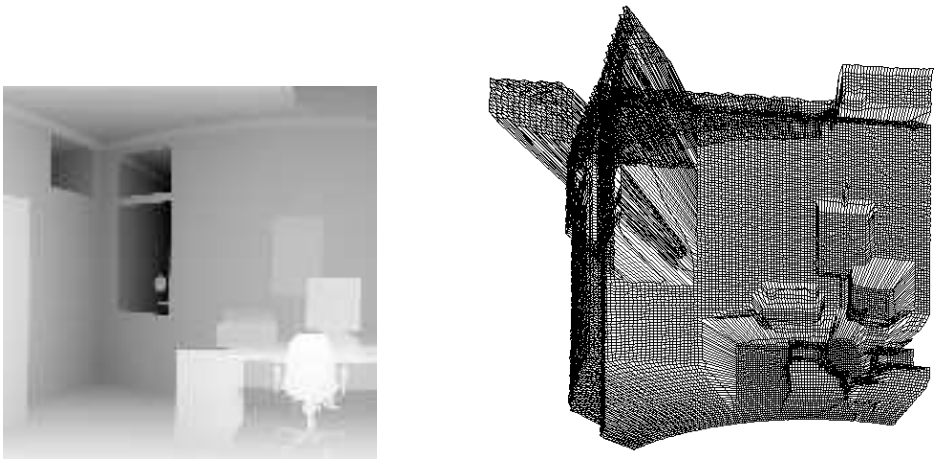
The range images used in the present work to create the 3D models of the environment are captured using a time-of-flight Laser Range Finder (LRF) mounted on a pan-and-tilt unit (two rotational degrees of freedom) on board of a mobile robot (see Fig. 2). Distances are measured on a grid with equally spaced pan and tilt angles. A typical image has 140 by 140 samples, covering a solid angle of 60° by 60°.



(a) (b)

Fig. 2. LRF: (a) on board of the mobile robot, (b) detail of the optical head.

The geometry of the scanning system introduces some geometrical deformations in the raw range image. Fig. 3 a) displays a grey level representation of a range image from a laboratory scene. Lighter areas correspond to shorter distances. It can be seen that the lines separating the wall from both the ceiling or the ground are curved. This illustrates the geometrical deformation inherent to the raw range image. A prior step to all processing is to correct the geometric deformations introduced by the scanning apparatus, by transforming the coordinate system of the range images. Raw range images are represented in a (quasi) spherical coordinate system using as variables the azimuth angle θ, the elevation angle φ, and the distance to the target $r(\theta,\varphi)$. The correction procedure generates a range image in a Cartesian coordinate system using the world coordinates x,y and z. A 3D view of the same range image with geometrically corrected data is shown in Fig. 3 b).



(a) b)

Fig. 3. Range Image from an office scene: a) grey level; 3D perspective view

Considering that a range image is used to represent a particular environment, special attention must be given to the selection of the parameters that characterise the acquisition procedure. Indeed, the final quality of the results at the end of the processing chain is heavily influenced by the raw image that is acquired. The following aspects should be considered:

- image field of view: scanning angle in both the horizontal and vertical directions;
- the area to be covered in a scan;
- image resolution: the angular distance between two consecutive samples;
- measurement time. The accuracy, statistical fluctuation and resolution of the distance measurements are dependent on the measurement time;
- the footprint dimension of the laser beam used;
- the scanning speed.

The above parameters are not totally independent, with some of them determining the others. As an example, it is not possible to have a totally independent choice of the measurement time, image resolution and scanning speed.

Several sources introduce errors in distance measurements. Some errors are inherent of the LRF, such as measurement noise or limited measurement resolution (<≈2 cm) and spatial resolution (the LRF uses a laser beam with 4 cm diameter and a divergence of 0.16°), others are inherent to the scanning system [15]. A major effect that distorts the image at specific locations (occluded edges) is the *mixed point* problem, where the *laser footprint* lies on two surfaces. In this case the range measurements are a weighted average of the two surface distances, having no physical meaning.

3. 3D SURFACE RECONSTRUCTION FROM A SINGLE VIEW

The problem of surface reconstruction from range data has been the focus of much research in the past decade [2-14]. Existing methods can be broadly classified into two categories:

i) global methods,
ii) local methods.

Methods are global in the sense that, in order to obtain the final reconstructed surface, linear or non-linear systems which involve all input data points have to be solved. Therefore, all data points are engaged in the shape definition at every location regardless the distance between them. In local methods, on the contrary, the definition of the surface shape at a given location depends only on the neighbourhood data. The global methods can again be divided into three subcategories:

i) variational principle based methods,
ii) probabilistic methods,
iii) deformable model.

Methods in the first subcategory formulate the surface reconstruction as an optimisation problem and solve it based on the variational principle [2,3], with global smoothness constraints introduced in the optimisation. Szeliski, [4], proposed an alternative formulation for the surface reconstruction problem based on the Bayesian framework. Geiger and Girossi, [5], considered a special case of this approach: the Markov Random Fields (MRF). Drawbacks of the MRF models are the computational complexity and the difficulty in estimating the parameters of the model.

The concept of a deformable model was first introduced by Terzopoulos et al, [6], to reconstruct approximately symmetric shapes from intensity data. Afterwards, some researchers have used this approach to reconstruct surfaces from range data. In general, deformable models can be divided into: elastically deformable models [7,8,9] and geometrically deformable models [10,11]. In elastically deformable methods, the basic surface description is a free-form deformable surface subject to external, internal, and inertial forces that follows the law of a mechanical system. In geometrically deformable models, the surface reconstruction is formulated as a cost function minimisation problem. The deformation of the surface is also subject to different types of forces which are cast as cost functions. Instead of tracing the dynamic surface deformation process, these models try to find the final deformed surface directly.

Among the global methods, those based on the variational principle and the probabilistic methods are often used to reconstruct surfaces from sparse data such as that coming from a stereo system. Because finite element techniques and MRF models are respectively used in these two methods, the reconstructed surface is represented in terms of a dense range image. Such a description can not be directly used for Computer Aided Geometric Design (CAGD) based applications where a mathematical representation of the surface is needed. Global methods, based on a deformable model, can on the contrary construct such a mathematical representation.

A drawback of the global methods is that they may execute unnecessary computation for data integration. Due to their global nature, each time some data is added (or extracted), the whole system must be solved in order to attain its new equilibrium state, no matter how local the information is. In addition, surface description resulting from such methods cannot represent the local details of object shapes, because this may require locally more energy, and the energy distribution is governed by a global system.

In contrast, local methods naturally lead to a piecewise surface description which provides a simpler mechanism for data integration and are more flexible in representing subtle object shapes. Due to the local nature of their construction process, each time a piece of information is added at some location, only the surface description near this location needs to be modified. Furthermore, it is possible to arbitrarily bend a surface patch to represent the local subtleties of object shapes without changing other parts of the surface description because there is no global system which couples them together.

Generally, local methods construct the surface description by first segmenting range data into regions and then fitting each segmented region with a surface primitive. The surface primitives more often used are quadratics [12] and superquadratics [13,14].

Based on these criteria it is considered that local methods are better suited for the present application considering that most objects, either in indoor or in outdoor urban environments, can be represented by planar patches bounded by linear edges. Furthermore, reconstructed models must extract meaningful information from noisy sensor data while making the weakest possible assumptions about observed shape. It is also desirable that the reconstructed surface description be as simple as possible while preserving its precision. If a piecewise surface representation is used, the number of reconstructed patches should be as small as possible, and if polynomial patches are used the degree of the polynomial should be as low as possible. The surface representation proposed in this work admits planar and biquadratic surfaces.

3.1 Geometrical Feature-Extraction

Once the range image has been acquired and geometrically corrected, it is possible to start the extraction of geometrical features that characterise the environment. These features are basically edges and surfaces. Edges correspond to surface discontinuities and can be divided into two categories: jump edges and crease edges.

The detection of surfaces delimits a number of adjacent 3D pixels belonging to a common surface. Typically a planar or a bi-quadratic surface. Edges and surfaces are complementary in the sense, that edges delimit surfaces, and the intersection of surfaces are edges. To construct a geometric model, at least one of these type of feature must be extracted directly from the raw data set. Traditionally, segmentation algorithms of range images, are based upon either a region approach or an

edge approach. The complementary of the two approaches predicts the advantage of a hybrid segmentation method, in which each approach compensates for the drawbacks of the other.

Two different approaches were implemented in this work: a edge-based (the basic idea is to fit pre-defined edge models to data, and then make a decision to either accept or reject the presence of such models [15]), and a hybrid algorithm that combines edge detection and region growing approaches to range image segmentation [16]. The edge detector method has good localisation properties but yields noisy edges. These are detected without pre-smoothing using geometric model fitting to obtain accurate edge location. Region growing gives stable surfaces but inaccurate border partitioning. The combination of both methods gives stable surfaces, eliminates internal microedges and provides for missing border reconstruction leading to a more reliable representation. The output of this hybrid approach is a segmentation map and an edge map. In addition, the edge map conveys C^0 (jump) and C^1 (crease) discontinuity information, and the segmentation map provides information on adjoining surfaces that resulted in the formation of edges and defines boundary conditions. This information will be used for occlusion detection. The final model admits planar and bi-quadratic surfaces. Fig. 4 is an is an overview of the hybrid algorithm.

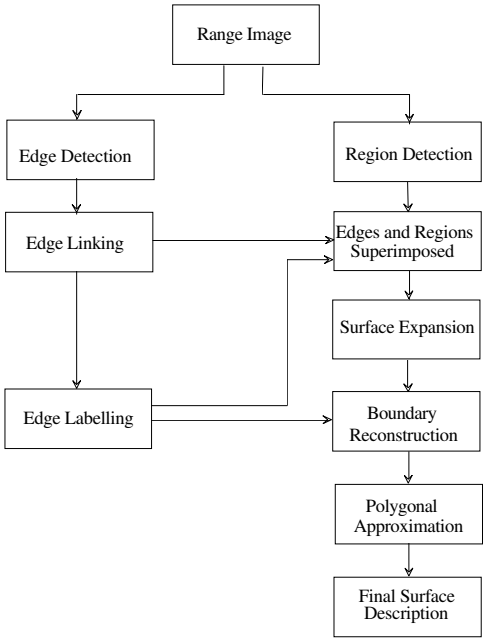
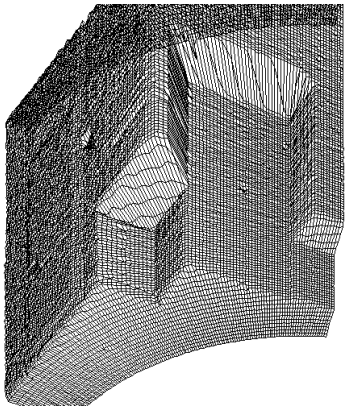


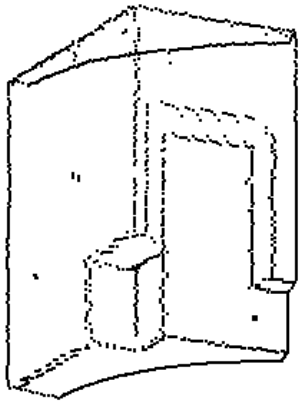
Fig. 4. Flowchart of the hybrid segmentation algorithm.

Fig. 5 and 6 present two examples of surface and boundary extraction from real world scenes. It can be seen in Fig. 5b that both jump and crease edges were well detected. However, as it is displayed, some of the jump edges are very noisy (see top of the cupboard in (b)) due to the mixed point effect. Final boundary reconstruction and surface representation obtained by the hybrid algorithm are displayed in (e) and (f) respectively. Fig. 6 shows the surface extraction from a complex scene with multiple objects and significant occlusions, both planar and non-planar surfaces (e.g., chair) are present. Different size objects are presented on this scene: an electrical box on the wall and a tube with the electrical cables, a table with a workstation and a printer, some drawers, a chair, half of a cupboard, a glass door, and a door with a glass window on top. The shadows on the upper left of Fig. 6 (a) are due to the glass door and the glass window. In fact the laser beam is not reflected by the glass, though the range measurements refer to the wall and part of the ceiling of the corridor and adjacent office. These areas had been clearly extracted and are represented in Fig. 6 (e) and (f).

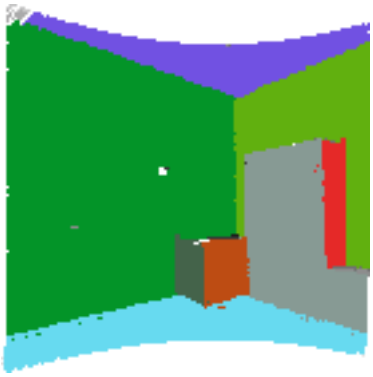
Different display perspectives were used to clarify the relevant aspects of each figure. The final surface representation contains explicit information about the boundaries of the segmented data. The good localisation of the edge detection scheme enhances the accuracy of the constructed boundaries. The white areas on the final representation are caused by the range shadows (occlusions) where no useful information is available. The algorithm presented in this Section has been successfully tested in other environments, with results reported in [16].



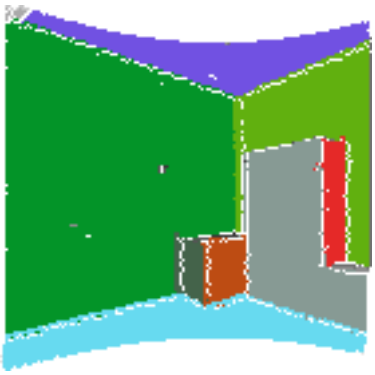
(a) 3D range image



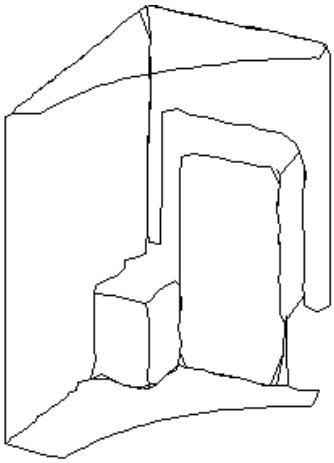
(b) 3D edge detection



(c) initial surface segmentation



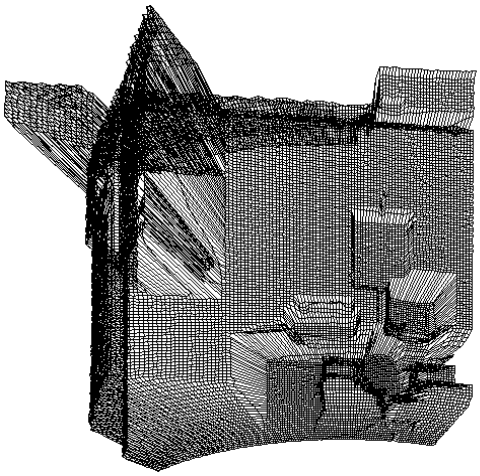
(d) edge and surface maps superimposed



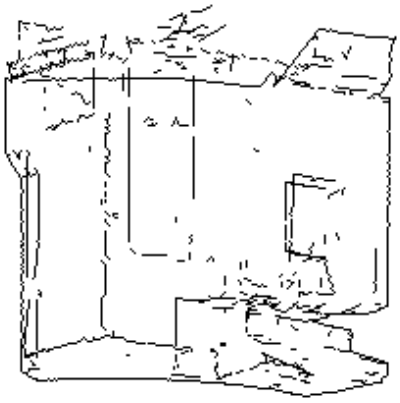
(e) 3D boundary reconstruction

(f) final 3D surface description

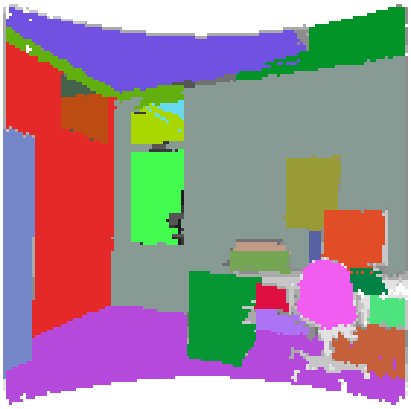
Fig. 5: Surface representation from a single range view.



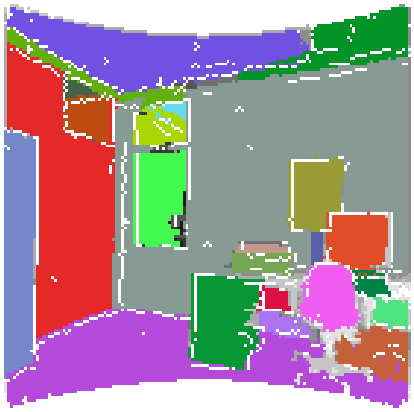
(a) 3D range image



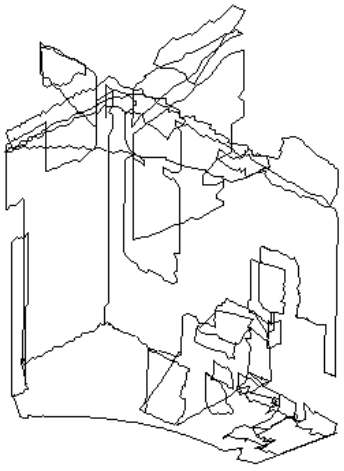
(b) 3D edge detection



(c) initial surface segmentation



(d) edge and surface maps superimposed



(e) 3D boundary reconstruction

(f) final 3D surface description

Fig. 6: Surface representation from a single range view.

4. 3D SCENE MODELLING FROM MULTIPLE RANGE VIEWS

Combining range images taken from multiple viewpoints into a single representation is essential for building a complete 3D model of a scene. Multiple range images from different viewpoints are acquired to resolve the ambiguities caused by occlusions and/or to introduce higher resolution images in region of interest. There is hence a need to:

i) detect the occurrence of occlusions;
ii) compute the next most appropriate capture point to resolve the occlusions;
iii) merge two or more range images from a single environment;
iv) integrate the geometric features found in each individual image;
v) build a single model for the whole scene irrespectively of the number of range images used to characterise the environment.

An important step in combining range images is the registration of partially overlapping images. The goal of registration is to find the transformation between the data from the different views. In the present case, the problem is registering two partially overlapping surfaces that may lack significant surface features. It is also required that the registration be as precise as possible. Based on those criteria, a hierarchical registration scheme working at the pixel level was implemented for higher accuracy.

Two different high-level descriptions of the 3D merged scene have been investigated: a surface description in terms of planar and bi-quadratic surfaces and respective 3D boundary representation, based on the surface descriptions from the single views, and a polygonal triangular mesh based on the range edge segments. These approaches are described in sub-sections 4.2 and 4.3.

4.1 Registration of Range Images

In general, registration can be done with or without correspondence of features. For systems based on feature correspondence, the accuracy depends mostly on the position accuracy of the used features, and is often not good enough for model construction. A different approach to registration followed in this work is directly supported on range data to iteractively find a 3D transformation which minimises a distance measured between two geometric shapes being registered [15].

The iterative algorithm assumes that an initial estimate of the 3-D rigid transformation between two partially overlapping range images is available. It can be obtained by a high level registration process or provided by the data acquisition process. Fig. 7 represents the registration scheme.

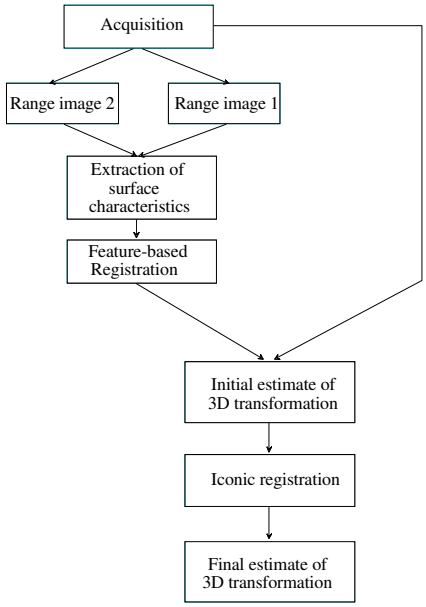


Fig 7: Registration of two overlapping range images.

The iconic registration algorithm is divided into two different steps:

i) Search of the corresponding point,
ii) Estimation of the 3D rigid transformation.

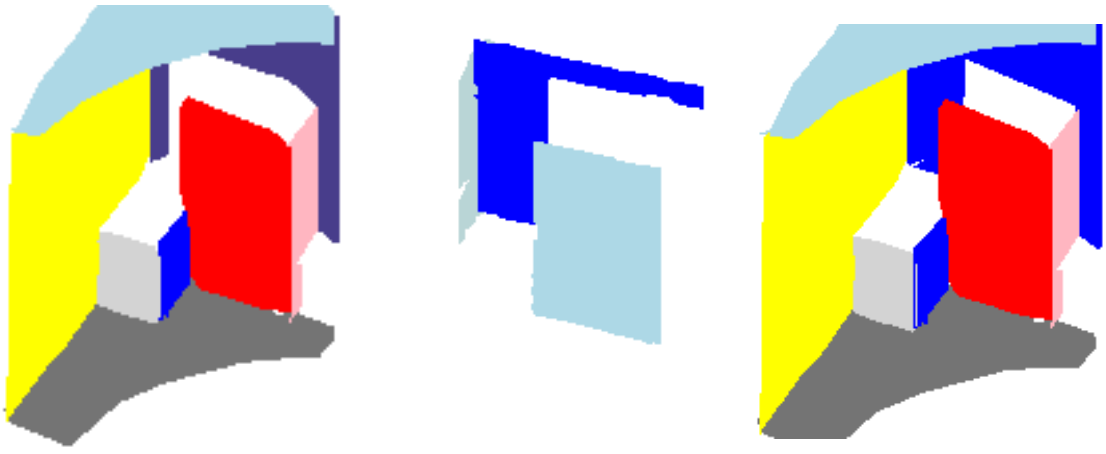
The problem of finding the nearest point, **q,** on a geometric shape Q to a given point **p,** has been discussed by Besl and McKay [17] (ICP algorithm), where Q can be represented by a set of segments, implicit curves, parametric curves, triangular faceted surfaces, implicit surfaces or parametric surfaces. In the present case, Q is a digital surface organised in a range image. If Q is big, the exhaustive search for the nearest point would be very time consuming. Another approach to the search problem followed in this work is to relax the demand for finding the exact nearest point **q** on Q to **p** [15,18]. Unlike most other registration techniques, this approach does not require point to point correspondence, since it minimises the distance from points to planes. Details about the algorithm can be found in [15].

Given a set of corresponding points, the least squares rotation matrix and the translation vector are estimated using the dual number quaternions [19]. The problem is formulated as an optimisation problem using dual-quaternions to represent translation and rotation. The advantage of using dual number quaternions instead of the normally used singular value decomposition (SVD) technique [20] or the quaternion method [21], is that the orientation and the position estimate are simultaneously solved by minimising a single cost function associated with the sum of orientation and position errors.

4.2 3D Scene Modelling using Geometrical Features

After the registration of neighbouring views, a surface description using the hybrid algorithm is constructed for each view, as described in Section 3.1. The surface descriptions of the different views are then transformed to a common coordinate frame. For constructing the model resulting from the combination of the multiple views, an average of the samples that are in the overlapping surfaces of the views is performed [16]. The surface patches that overlap are detected and then the surface parameters that fit to the fused data are adjusted. The merged surface is then expanded to the edge points, that were previously merged, and the boundary is reconstructed. The integration of surface descriptions can be summarised as follows:

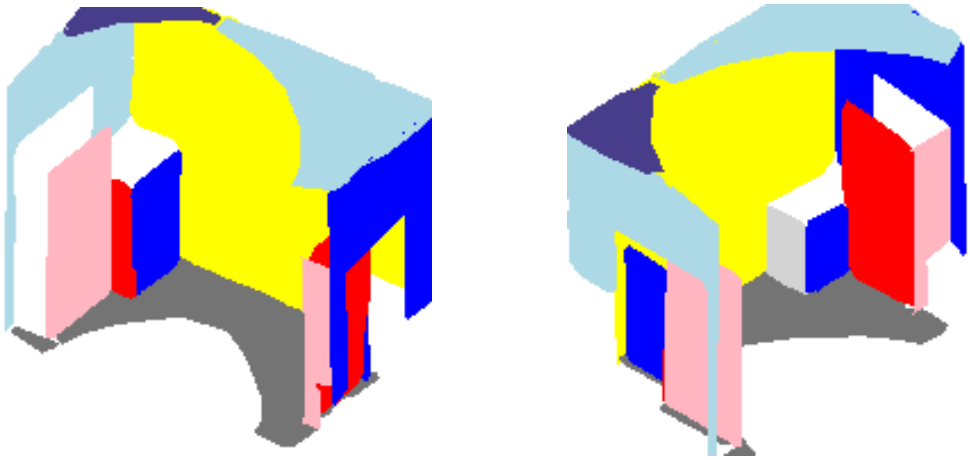
i) Construct a surface description for each view using the hybrid algorithm.
ii) Find overlapping surfaces. The surface descriptions of the different views were previously transformed to a common coordinate frame using the 3D rigid transformation obtained from the registration process.
iii) Merge points from the partially overlapping surfaces.
iv) Refit a surface model to the merged surface points.
v) Fusion of edge points. Expansion of the merged surface to the edge points.
vi) Boundary reconstruction.



(a) 3D model with occluded area (b) 3D patch model (c) final representation

Fig. 8: Elimination of occlusion area

The region fusing procedure enables the elimination of occluded areas (or areas with noisy/ambiguous data in the raw range image) in the 3D model extracted from a single range image (Fig. 8a) by fusing another model extracted from a range image of the occluded area (Fig. 8b), into a single model (Fig. 8c). The range image in Fig. 8b was acquired with higher resolution to increase region boundary accuracy. The present integration scheme is powerful in that the combination of views may be done at the level of the final representation. A complete 3D surface model resulting from the integration of three range images is presented in Fig. 9.



(a) (b)

Figure 9: 3D surface model extracted from three range images of an office scene: (a) left view, (b) right view.

4.3 3D Scene Modelling using Triangular Faceted Surfaces

The scene reconstruction process starts from range segments, and is based on a 2D constrained Delaunay triangulation [15]. This is done in the image plane and results in a triangular faceted piecewise linear description of scene surfaces. Since the input data is a range image, a 2D triangulation method can be used to construct a surface triangulation. The result of the whole procedure is a triangular-faced piecewise linear surface, in which segments are preserved. An interesting feature of this approach is its fairly low computational cost, considering that most of the processing is done in 2D. The surface reconstruction procedure consists of three phases:

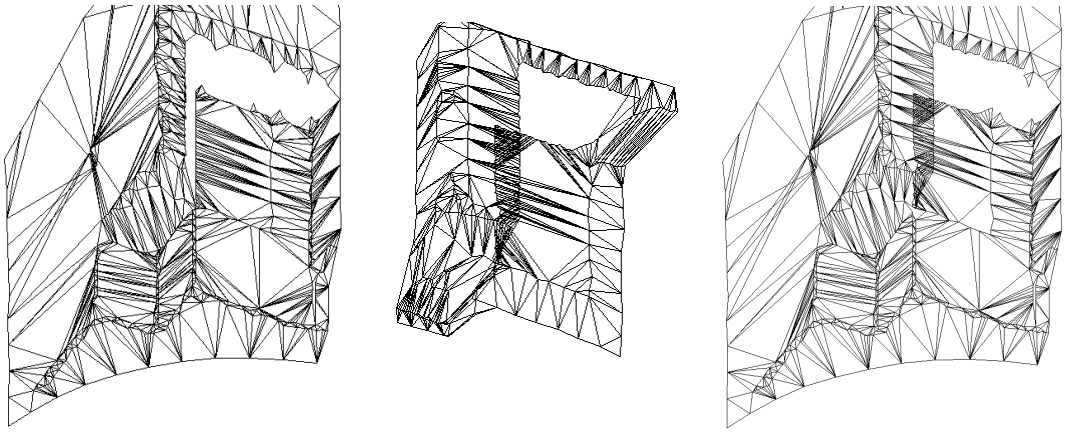
i) Reconstruction of range segments;
ii) Constrained Delaunay triangulation in the image plane;
iii) Back projection of the 2D tessellation onto the 3D space.

The integration of different views is based on a contour propagation technique. After having registered two neighbouring views, an initial surface description, represented as a polygonal triangular mesh is constructed from one view. The bounding contours are then transported to the second view using the 3D rigid transformation obtained from the registration process[22]. These transported contours are embedded as constraints in the surface description of the second view.

In order to illustrate the integration of the previously registered images, the merging of two views described as triangular-faced piecewise linear surfaces into a single 3D representation is shown in Fig. 10. 3D edge segments were first extracted from each image using the algorithm described in Section 3.1, and a curve was traced on the overlapping part of the two images. This curve was then mapped onto the second image coordinate system using the 3D rigid transformation obtained from the registration process. Triangular based surface descriptions were then constructed for both images, with the traced curve and its transformation embedded in the two surface descriptions. A 3D shaded model of an office scene, resulting from the integration of surface descriptions from three views, is seen in Fig. 11.

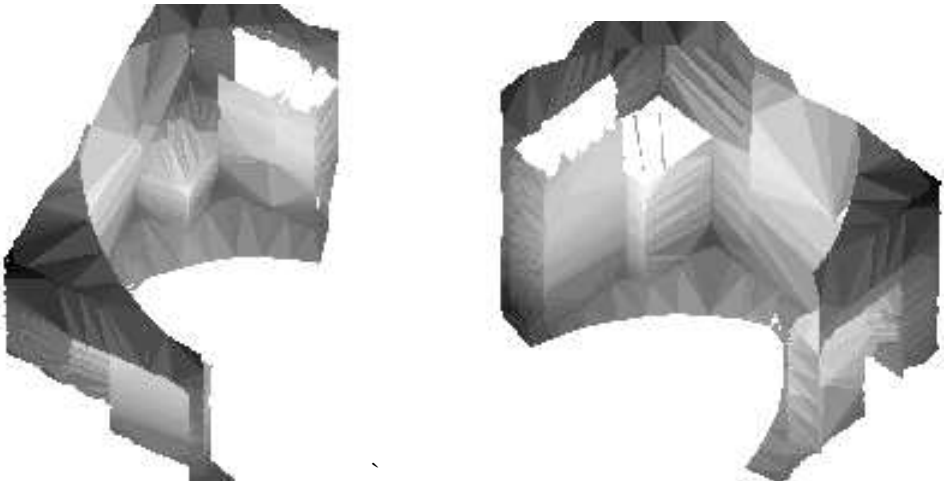
5. EXPERIMENTAL RESULTS FROM A LABORATORY ENVIRONMENT

A complete 3D surface model resulting from the integration of 5 range images of a laboratory (scanned area of 10x4 m, see Fig. 2 and 3) is presented in Fig. 12. This example is a sub-set of the full room description which has been reconstructed from 12 images. The scene was reconstructed using the technique described in Section 4.2. The final result is a 3D model of



(a) (b) (c)

Fig. 10: Merging of two surface descriptions into a single 3D representation (a,b) domain triangulation with embedded contours (c) integrated view.



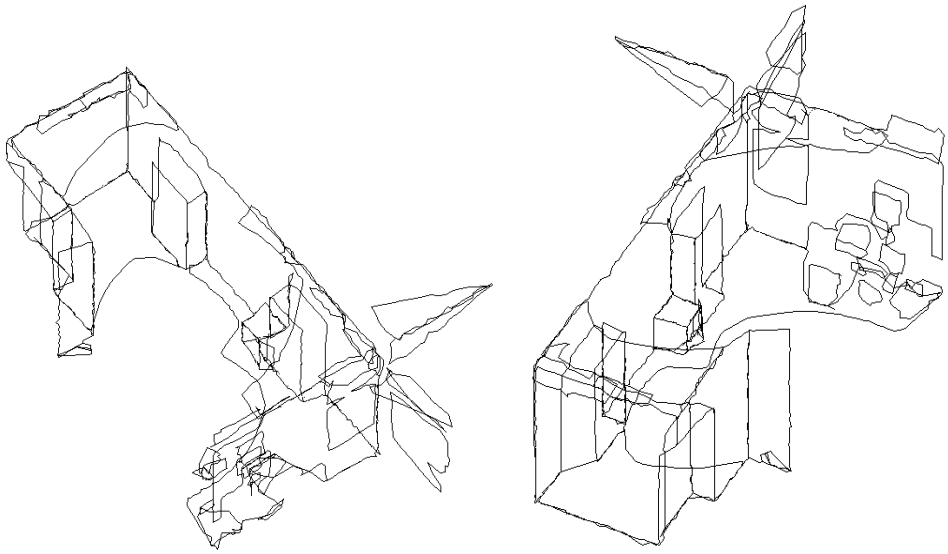
(a) (b)

Fig. 11 3D shaded model of an office seen: (a) right view (b) left view.

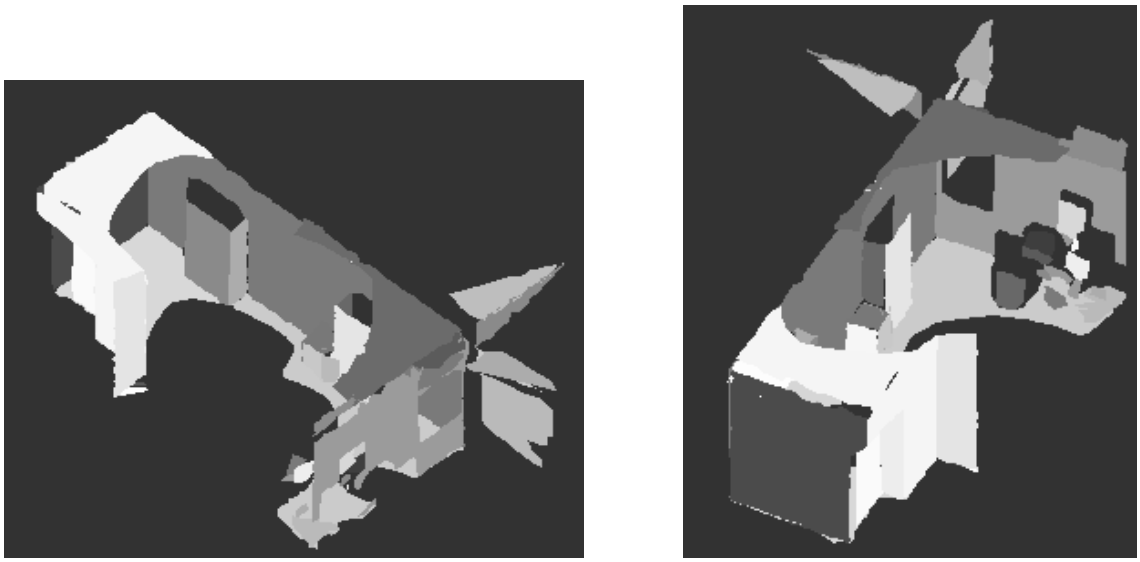
the laboratory described in terms of planar and bi-quadratic surfaces with explicit boundary information. Range images with different spatial and range resolutions were acquired using the scanning mechanism described in Section 2, from 5 different positions of the mobile robot. The reconstruction was achieved without any human intervention to correct the final model. Algorithm's thresholds have been kept constant for the processing of the different range images, this proving the robustness of the algorithms due to the presence of different size and shape objects. It should be stressed that reconstructed models must extract meaningful information from noisy sensor data while making the weakest possible assumptions about observed shapes.

6. CONCLUSIONS AND WORK IN PROGRESS

This paper describes a new 3D scene analysis system that automatically reconstructs the 3D geometric model of real-world scenes from multiple range images acquired by a Laser Range Finder on board of a mobile robot. It describes an integrated procedure for the automatic acquisition of CAD models of already built environments, that includes range data acquisition, geometrical feature extraction, occlusion detection, registration and integration of multiple views into a single representation.



(a) boundary after merging



(b) final surface description

Fig. 12: High-level surface description from composite range images

The novel contributions of the paper are:

- Build 3D models of real-world environments (distance range from 0.5 to 20 meters) from range images acquired by a Laser Range Finder on board of a mobile robot.
- A new hybrid algorithm for the segmentation and representation of noisy range images, combining edge and region detection techniques. This algorithm gives better results, i.e. more consistent with geometrical features present in real world scenes (tested on a large number of range images), than traditional edge or region detection algorithms.
- Reconstruction of a consistent 3D representation of surface regions and discontinuities.
- Fusion of multiple range images into a single consistent 3D model.
- Fully integrated procedure from range acquisition to the final model including occlusion detection, and its resolution.

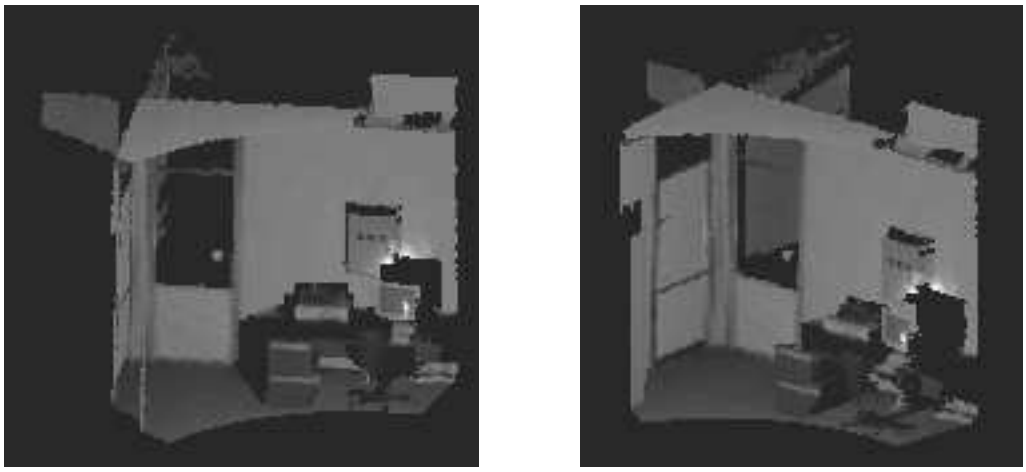


Fig. 13. 3D model with texture information from reflectance image.

Some of the main features of the current implementation that enhance its novelty are summarised:

- Range image creation using active range sensing (time-of-flight range finder) rather than passive techniques (e.g. shape from motion or stereo).
- Fully automated model reconstruction without any manual intervention to correct the final model.
- No a priori information about the environment (e.g., shape, size) is used for the modelling process.
- It is not assumed any particular orientations for the extracted edges, and both bi-quadratic and planar models are presented in the final representation.
- The algorithm for edge detection works on the 3D space based on geometric model fitting, instead of traditional 2 1/2 D.
- Both algorithms for edge and region detection work on sparse 3D points rather than on an equally spaced grid.
- Integration of edge and surface representations is done in 3D Cartesian coordinates.
- Final surface boundary is constrained by the edges and the surface parameters result from the region detection.
- The final models contain explicit information about the boundaries of the segmented data.

To select the next viewing direction the perception planning process, taking into account the constraints of having sufficient overlap between views to allow registration, estimates the viewing direction that, for the unknown data, gives the maximal amount of new information regardless the final number of views. Strategies for optimally selecting capture points aiming at minimising its number are currently under study.

The iterative procedure for 3D reconstruction presented in this paper has been tested successfully in several different environments, demonstrating the use of laser range sensing for 3D scene modelling of pre-existing environments.

Current work is being made to integrate other sensory information such as the reflectance image to make the overall process more robust and reliable, and to have more realistic models by rendering the geometric models with natural texture. Preliminary results using the texture information from the reflectance image are shown in Fig. 13. This is a new concept called virtualized reality, that turns a real world scene into a virtual one. This differs very significantly from traditional virtual reality, in the sense that instead of generating a virtual world, described using CAD models from computer graphics, it starts with the real world and virtualize it.

The resulting 3D scene description is directly converted to the emerging VRML (Virtual Reality Modelling Language) 3D format, and can also be directly converted to a CAD file format (e.g., DXF) so that it can be manipulated or displayed using conventional CAD programs.

The developed technique can be used to support the following application areas:

i) Retrieval of architectural and/or industrial plans into a CAD model.
ii) Design verification of buildings and the corresponding plans.
iii) Navigation of autonomous robots.
iv) Virtualized reality.

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