A PROBABILISTIC APPROACH FOR THE LOCALIZATION OF MOBILE ROBOTS IN TOPOLOGICAL MAPS

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Abstract

The navigation of mobile robots in outdoor environments is becoming increasingly important fostered by a large number of challenging applications. Localization, map building and world representation are key issues for the navigation in unstructured environments. This paper presents a probabilistic approach for the localization of mobile robots aiming at outdoor applications. A Markov model and a topological map for world representation are the frameworks that support the proposed algorithm. Simulation results illustrate the performance of the localization procedure.

1 Introduction

Mobile robots navigation, in particular in outdoor environments, mimics, in a way, the human behavior when subsisting in such environments. Environment perception is the main procedure that links the actor (human or robot) with the surrounding world. In addition, human beings create a particular world representation and integrate their own observations with the already acquired/learnt map to ensure local positioning at a given time instant this permitting path planning, i.e., the decision on how to reach a desired location. Environment perception enables map building or map enhancement. The map, which is a world representation, collects the key features extracted from the information acquired by the sensors.

This paper presents a methodology for mobile robot localization in outdoor environments using a topological map that collects the main features of particular places in the world. A probabilistic approach based on Markov Models is the framework that supports the robot's localization and accounts for observation uncertainty.

The use of topological maps permits the exploration of unknown and unstructured environments in the absence of metric information, mainly in outdoor environments. In this type of map, world is represented by graphs (not necessarily 2D) composed by a fixed number of discrete places linked by bidirectional paths and/or actions. This world representation combines the optimal tracking capabilities of feature maps – featured base mapping – with the scalability of a topological map. The map used in the present work is structured as a topological graph of nodes and edges.

A similar world representation is described in [2], where the author proposes a topological featured map. The main goal is to facilitate the implementation of large-scale Simultaneous Localization and Map building (SLAM). The map is also a topological graph of nodes and edges. Each node, which defines a unique physical location, is represented by a set of nearby features. The edge connecting two nodes specifies the coordinate transformation between the poses of the corresponding nodes.

The advantages of topological maps representation for robot localization are scalability, incrementally built/update, no map matching and simultaneously reducing the uncertainty of localization and perception of the environment. The methodology presented in this paper for localization does not require the exact position (geometric position) of the robot which enables the use of the topological map as a world representation. The map resolution is proportional to the complexity of the environment representation. Compactness is a key advantage of this type of representation that allows fast planning and facilitates interfacing to symbolic planners and problem-solvers. The disadvantages include, for some applications, the absence of metric information and the requirement of feature (or landmark) selection/detection/recognition. This also means that topological representations are heavily dependent on a powerful system to identify elements of the environment. As a result, one of the most known localization problems using a topological representation occurs when the robot traverses two places that look alike. The topological approaches often have difficulty determining if theses places are the same or not (particularly if these places have been reached via different motion commands, actions or paths), as mentioned in [9].

The localization problem solved with a probabilistic approach is robust in face of sensor limitations, sensor noise and environment dynamics as referred in [8], where the author identifies the global localization problem as particularly challenging. Using a probabilistic approach, the localization turns into an estimation problem with the evaluation and propagation of a probability density function, see [5]. Within the context of mobile robot localization, this type of approaches are often referred to as Markov Localization (ML) or Hidden Markov Models (HMM) as shown in [7] and [8], respectively.

The novelty of this paper is the localization of mobile robots in outdoor environments, using a probabilistic approach supported on Markov Models and designed on top of a topological map that collects environment features. The main tool used to accomplish the localization on this world representation is a modified version of the Forward-Backward algorithm, [6].

The paper is organized as follows. Section 2 presents an overview of topological world representations and introduces the notation of the map's parameters that will be used to support localization. The localization problem of mobile robots in outdoor environments, using topological world representations, is discussed in Section 3. Subsection 3.1 details the Markov Model as a probabilistic approach, where the Forward-Backward (FB) algorithm is mentioned and revisited in Subsections 3.2 and 3.3, respectively. Experimental simulated results are presented in Section 4. Section 5 concludes the paper and presents directions for further developments.

2 Topological Maps

A map is a representation of (part of) the environment's surface (even tri-dimensional shape) showing physical (natural or artificial) features that characterize particular locations or places. In a topological map no metric representation is available. Instead, the map expresses a functional relationship among relevant features with a resolution that is proportional to the complexity of the environment representation. The main structure of a topological map is composed by a set of nodes that, in this work, represent places in an outdoor environment, and edges that link the nodes and account for the physical or logical relationship between the nodes. In the case where the topological map supports the navigation of a mobile robot, edges may represent the actions taken by the robot to travel between the nodes.

In the present work, a topological map supports the navigation of a mobile robot, in particular its localization. In this context, the state of the robot is related with the environment places in its neighborhood which justify the introduction of the notion of map state. Each node is defined as a state of the map. Each state is characterized by a set of relevant features to support the state identification and to avoid mismatching. Similar features are grouped in the same attribute set.

The notation used to define a topological map is the following:

- s_i is the state i of the map,
- $S = \{s_1, s_2, \ldots, s_N\}$ is a set of N states of the

map,

- v_j is the j^{th} feature or attribute, $j = 1, \ldots, M$, that may classify any state s_i,
- $v_j \in V_j$, i.e., the j^{th} attribute takes values in the set V_j,
- $s_i(v_j)$ is the value of the attribute v_j at state s_i,
- $s_i(v_j) = \emptyset$ means that the attribute v_j is unknown at state s_i.

In a traditional map of a city, the topological representation is often used as a support for path planning. The relevant interesting places, considered as map states, are emphasized with the most important features like buildings, monuments, shops, stations. The states are linked by roads, streets and rails. This symbolic representation requires well characterized states and a good recognition to avoid mismatching places that look alike. An example of a topological representation of the map of a city is represented in Figure 1 where seven different states ($S = \{s_1, s_2, \ldots, s_7\}$) are displayed. Three features (v_1, v_2, v_3) are used: transport, building and leisure. The values taken by each feature are, for example, $V_1 = \{$airport, underground, boat, train, parking$\}$, $V_2 = \{$university, castle, church, statue$\}$ and $V_3 = \{$camping, garden, restaurant$\}$.

The main goal of localization, in the context of robot navigation, is related with identifying the state of the map in the closest vicinity of the mobile robot, which is a concept that will be formalized in Section 3. To perform localization the robot perceives the environment with its on-board sensors and the acquired data is processed aiming at extracting the most relevant features and localizing the robot by a matching with the real features. This points to the key role played by the sensing processing that collects observations, for which the following related notation will be used:

- $o_t = [\ o_t(v_1)\ \ o_t(v_2)\ \ \ldots\ \ o_t(v_M)\ ]$ is a M-dimensional observation vector at time instant t,
- $o_t(v_j)$ is the value of the attribute v_j extracted from the observation o_t,

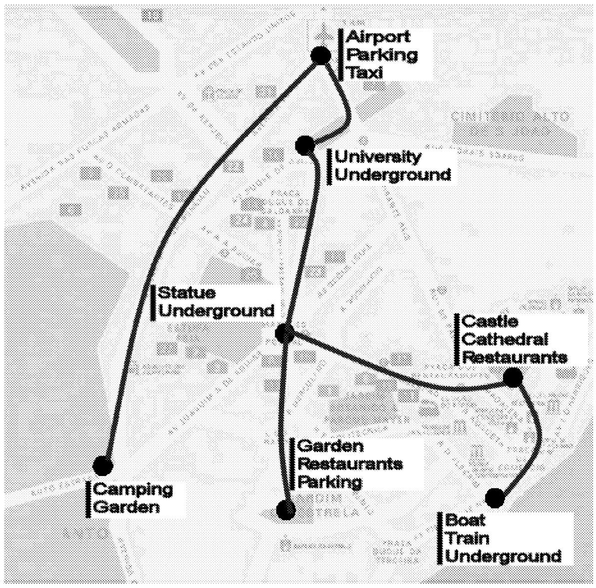

Figure 1: An example of a topological map in a city

- $o_t(v_j) = \emptyset$ means that observation of the attribute v_j at time instant t is not achieved,
- $O_t = \{o_1, o_2, \ldots, o_t\}$ is an observation sequence up to the time instant t.

As a result of the localization procedure, the current state of the mobile robot is evaluated. The following notation being used:

- q_t is the robot location in time instant t,
- $Q_t = \{q_1, q_2, \ldots, q_t\}$ is a state sequence up to the time instant t,
- $q_t \in S$.

3 Localization

The localization procedure proposed in this work states that, at each time instant, the robot location, q_t, is equal to the map's state in its closest vicinity using a probabilistic approach to decide on this proximity function. The robot estimated location is the map's state that is most likely to have produced the observations acquired by the robot sensors during a given time interval. This is notably different from the usual localization procedures that aims at providing a pose (position and orientation) estimation in a local or global frame. In fact, when the proposed localization procedure yields a robot estimated location $\hat{q}_t = s_i$ this does not mean that the robot physical location (pose) coincides with that of the environment place that lead to the map state s_i.

As a result of the measurements uncertainty, the current robot state has to be estimated. The main issue of the localization problem is to find the state that minimizes the uncertainty, given the observations. The state estimation at each time instant t is evaluated using all the available observations during the interval T, with $t \leq T$ which means that the time instant t is not necessarily the current time instant. According to a probabilistic approach, the current state estimation, $\hat{q}_t$, is the argument that maximizes the conditional probability density function (pdf) of the state given the observation sequence $O_T = \{o_1, o_2, \ldots, o_t, \ldots, o_T\}$ acquired in the time interval T, i.e.,

$$\hat{q}_t = \arg\max_{q_t} P(q_t = s_i \mid o_1, \ldots, o_T) \qquad (1)$$

Mobile robot sensors acquire raw data whose nature depends on the type of sensors. For example, vision cameras provide intensity images, while laser range sensors or ultrasound sensors provide range data. Irrespectively of the type of sensor, it is assumed that raw data is processed and that the relevant features are extracted. These features are those used as attributes for the characterization of the states of the topological map. In the context of the localization methodology described in this paper, (1), observations are considered to be the state attributes extracted from sensor data along the mobile robot trajectory.

To evaluate (1) it is necessary to compute the pdf of the state given the observation sequence. The following subsections introduce the Markov Models and the Forward-Backward algorithm that support the evaluation of (1).

3.1 Markov Models

The localization process requires the knowledge of the information $(o_1, o_2, o_3, \ldots, o_T)$ to solve (1) at time instant t, which is a rough problem. However, the strong Markov assumption, valid in this context, overcomes the problem.

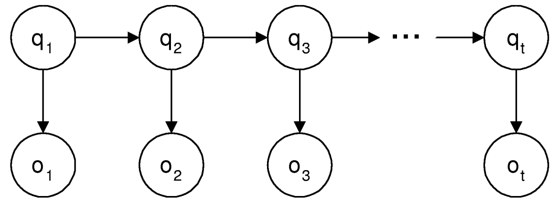

Figure 2: Markov Model: states and observations assumptions

The Markov property, [3], states that the knowledge of the state at time instant t, q_t, is enough to determine the pdf of the state in future instants, or, in different words, all the information acquired before the time instant t is already reflected in the state estimate q_{t-1}. In addition, the observation o_t depends only on the state of the mobile robot at the time instant t. This Markov property is graphically illustrated in Figure 2, where the variable q_t corresponds to the mobile robot state estimation at each time instant.

A Markov Model (MM) is the framework that supports the study of the state evolution along time. The MM is completely characterized by three pdf, (2), (3) and (4). The first one is the state transition probability density, a_{ij}, that defines the probability of changing from the state i to the state j, as illustrated in Figure 3. The state transition probability in the a_{ij} parameter characterizes the edge of the topological map between states s_i and s_j. The value of each transition may depend on the distance between the states but can also be acquired during a training phase which is updated along the navigation. The

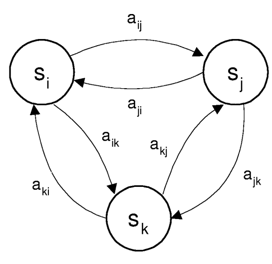

Figure 3: Markov Model with 3 states s_i, s_j and s_k and selected state transition pdf

second pdf that characterizes the MM is the observation probability density function $b_i(o_t)$, that represents the probability of observing o_t at the time instant t given that the state is s_i. Finally, the initial state distribution, π_i, that represents the probability that the robot initial state is s_i, this being defined *a priori*. The compact notation for the MM characterization is represented by $\lambda = (A, B, \pi)$, where A is a square matrix, containing all the probability transitions, $a_{ij}, i, j = 1, \ldots, N$, B is a vector of the observation probability in all possible states, $b_i(o_t), i = 1, \ldots, N$ and π is a vector that contains the initial localization probability in all possible states, $\pi_i, i = 1, \ldots, N$.

$$
\begin{aligned}
a_{ij} &= P(q_{t+1} = s_j \mid q_t = s_i), i, j = 1, \ldots, N \quad (2)\\
b_i(o_t) &= P(o_t \mid q_t = s_i), \quad i = 1, \ldots, N \quad (3)\\
\pi_i &= P(q_1 = s_i), \quad i = 1, \ldots, N \quad (4)
\end{aligned}
$$

The observation pdf depends on the sensors' model. In this work we assume that the sensors are modeled as a Gaussian, (5),

$$P(o_t \mid q_t = s_i) = N(o_t, \mu_i, R_i) \quad (5)$$

It is also assumed that each state is modeled as a sum of Gaussians, $\sum_{l=1}^{G} k_{il} N(o_t, m_{il}, P_{il})$, where m_{il} is the mean vector and P_{il} the covariance matrix of the state attributes. This model validity increases with the number of Gaussians, G.

Based on Markov Models, the localization procedure in (1) is similar to the high-dimensional maximum likelihood estimation problem, as referred in [6]. This problem is efficiently solved using the Baum-Welch algorithm, as well as the Forward-Backward (FB) algorithm or simply the Alpha-Beta algorithm. The same problem is referred in [10] as a special version of Expectation and Maximization.

3.2 Forward-Backward algorithm

The localization problem requires the evaluation of the argument that maximizes the pdf in (1). To accomplish this goal, the FB algorithm described in this subsection, is applied along the lines described in [6]. The conditional probability of the current state given the observation sequence during the time interval T is decomposed in two pdfs, $P(o_1, \ldots, o_t, q_t = s_i)$ and $P(o_{t+1}, \ldots, o_T \mid q_t = s_i)$, as represented in (6),

$$
\begin{aligned}
P(q_t = s_i \mid O_T) &= \\
&= \frac{P(o_1, \ldots, o_t, q_t = s_i) P(o_{t+1}, \ldots, o_T \mid q_t = s_i)}{P(O_T)} \\
&= \frac{\alpha_t(i) \beta_t(i)}{P(O_T)} \quad (6)
\end{aligned}
$$

where

$$
\begin{aligned}
\alpha_t(i) &= P(o_1, \ldots, o_t, q_t = s_i) \quad (7)\\
\beta_t(i) &= P(o_{t+1}, \ldots, o_T \mid q_t = s_i) \quad (8)
\end{aligned}
$$

One of the probabilities in (6), the $\alpha_t(i)$ parameter (7), is related with all the observations from the past up to the time instant t. On the other hand, all the future observations are in the $\beta_t(i)$ parameter, (8). Applying the Bayes law in the denominator of (6) and using (7) and (8), yields

$$
\begin{aligned}
P(q_t = s_i \mid O_T) &= \frac{\alpha_t(i) \cdot \beta_t(i)}{\sum_{i=1}^{N} \alpha_t(i) \cdot \beta_t(i)} \\
&= \eta_T \cdot \alpha_t(i) \cdot \beta_t(i) \quad (9)
\end{aligned}
$$

where η_T is a normalization factor.

The main result expressed in (9) is that there is a complete decoupling on the observation sequence relative to the time instant t. The observations prior to t (past observations) and the observations from t to T (future observations) appear in different factors. This decomposition is illustrated in Figure 4, where the α and β parameters are represented during a time interval T.

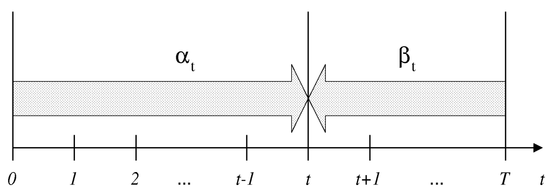

Figure 4: The past, α, and the future, β, influences the time instant t

Whenever t is the current time instant and $t \leq T$, it is reasonable to consider that there is no future observations up to T, and therefore $\beta_t(i)$ is considered to have a uniform distribution, this expressing the absence of future information. On the contrary, whenever the current time instant is t_1 with $t < t_1 \leq T$, the state estimate at time instant t can profit from the observations from t to t_1. This leads to increasing the quality of the α_t parameter in future localizations.

To evaluate the pdf (9) for each time instant t, it is necessary to evaluate $\alpha_t(i)$ and $\beta_t(i)$. The FB algorithm, [6], provides an iterative solution to this problem. The forward and backward iterations of the algorithm are represented in (10) and (11), respectively,

$$\alpha_{t+1}(j) = \left[\sum_{i=1}^{N} \alpha_t(i) a_{ij}\right] \cdot b_j(o_{t+1}) \quad (10)$$

$$\beta_t(i) = \sum_{j=1}^{N} a_{ij} \cdot b_j(o_{t+1}) \cdot \beta_{t+1}(j) \quad (11)$$

where a_{ij} and b_j are the parameters that characterize the underlying Markov Model. The reader should notice that (10) evolves forward in time while (11) evolves backwards. The forward and backward iterations are initialized at time instant $t = 0$ and $t = T$, respectively. The initialization of (10) requires *a priori* information in the variable π, as expressed in (12). The initialization of the variable β corresponds to an uniform distribution, as stated in (13),

$$\begin{aligned} \alpha_1(i) &= P(q_1 = s_i) \cdot P(o_1 \mid q_1 = s_i) \\ &= \pi_i \cdot b_i(o_1) \quad (12) \\ \beta_T(i) &= 1 \ , \ 1 \leq i \leq N \quad (13) \end{aligned}$$

3.3 Forward-Backward algorithm revisited

In the FB algorithm described in the previous subsection, the time interval of length T from $t = 0$ to $t = T$ has a fixed length, while t is the single variable of time. For long time intervals, corresponding to large operating periods, the FB algorithm implementation becomes too time consuming. For this reason, it is necessary to understand what has to be changed in the algorithm if the length of the observation sequence, T, is also considered to be a variable.

Rewriting the equations (7), (8) and the iterations, (10) and (11) of the previous section as a function of T, the results are expressed in (14), (15), (16) and (17). The main difference is the parameter representation, which includes the variable T in superscript.

$$\alpha_t^T(i) = P(o_1, \ldots, o_t, q_t = s_i) \quad (14)$$

$$\beta_t^T(i) = P(o_{t+1}, \ldots, o_T \mid q_t = s_i) \quad (15)$$

$$\alpha_{t+1}^T(j) = \left[\sum_{i=1}^{N} \alpha_t^T(i) a_{ij}\right] \cdot b_j(o_{t+1}) \quad (16)$$

$$\beta_t^T(i) = \sum_{j=1}^{N} a_{ij} \cdot b_j(o_{t+1}) \cdot \beta_{t+1}^T(j) \quad (17)$$

To evaluate $\alpha_t^T(j)$ and $\beta_t^T(j)$ as a function of T, consider, as an example, two distinct values for T, T_1 and T_2, with $T_1 \leq T_2$. As illustrated in Figure 5, $\alpha_t^{T_1}$ is equal to $\alpha_t^{T_2}$, for $t \leq T_1$, (18), since both consider the same past information.

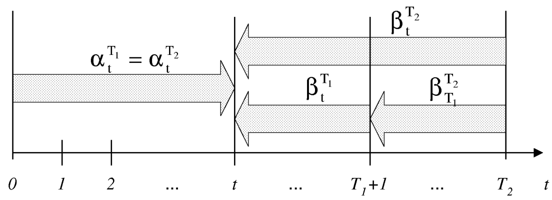

Figure 5: The past and future influence at time instant t. The future is divided in two sub-intervals

It is thus evident that,

$$\alpha_t^{T_2}(j) = \alpha_t^{T_1}(j), \qquad t \leq T_1 \quad (18)$$

while $\beta_t^{T_2}$ is decomposed in two parameters: the first contains the information of the interval from t to T_1 and the second results from the observations acquired during the fixed interval from T_1 to T_2,

$$\beta_t^{T_2}(i) = \beta_t^{T_1}(i) \cdot \beta_{T_1}^{T_2}(i) \quad (19)$$

Consider now that the overall observation sequence interval is sampled in time intervals of length T, $kT + T$, as illustrated in Figure 6. Using the introduced notation, this yields:

$$\begin{aligned} O_{kT+T} &= \{o_1, \ldots, o_{kT}, o_{kT+1}, \ldots, o_{kT+T}\} \\ &= \{O_{kT}, o_{kT+1}, \ldots, o_{kT+T}\} \quad (20) \end{aligned}$$

With the definition $O_{kT}^{kT+T} = \{o_{kT+1}, \ldots, o_{kT+T}\}$ the observation sequence up to $kT + T$ is written as

$$O_{kT+T} = \{O_{kT}, O_{kT}^{kT+T}\}$$

The probability of the current state given the observation sequence, $\{o_{kT}, O_{kT}^{kT+T}\}$, along the acquired time interval from kT to $kT + T$ is given by

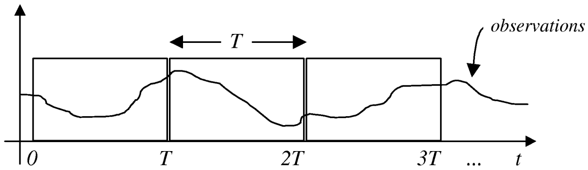

Figure 6: Observation sequence divided in time intervals of length T

$$
\begin{aligned}
&P(q_t = s_i \mid O_{kT+T}) = \\
&= P(q_t = s_i \mid o_1, \ldots, o_{kT}, \ldots, o_{kT+T}) \\
&= P(q_t = s_i \mid o_{kT}, o_{kT+1}, o_{kT+2}, \ldots, o_{kT+T}) \\
&= P(q_t = s_i \mid o_{kT}, O_{kT}^{kT+T}) \\
&= \frac{\alpha_t^{kT+T}(i) \cdot \beta_t^{kT+T}(i)}{P(O_{kT+T})} \qquad (21)
\end{aligned}
$$

where α and β are expressed in (22) and (23), respectively. Both equations require that $kT \leq t \leq kT + T$.

$$
\begin{aligned}
\alpha_t^{kT+T}(i) &= P(o_{kT}, \ldots, o_t, q_t = s_i) \qquad (22) \\
\beta_t^{kT+T}(i) &= P(o_{t+1}, \ldots, o_{kT+T} \mid q_t = s_i) \qquad (23)
\end{aligned}
$$

To evaluate the pdf (21) for each time instant t, it is necessary to evaluate $\alpha_t^{kT+T}(i)$ and $\beta_t^{kT+T}(i)$. As described in the previous subsection, it is also necessary to apply the forward and backward iterations. At each interval of length T, the initializations of both parameters are required. The initialization of (22) and (23) are presented in (24) and (25), where it is used the fact that the probability $P(q_{kT+1} = s_i)$ is the same as $P(q_{kT} = s_i)$.

$$
\begin{aligned}
\alpha_{kT+1}^{kT+T}(i) &= P(q_{kT+1} = s_i) \cdot P(o_{kT+1} | q_{kT+1} = s_i) \\
&= \pi_i^{kT+T} \cdot b_i(o_{kT+1}) \qquad (24) \\
\beta_{kT+T}^{kT+T}(i) &= 1 \ , \ \leq i \leq N \qquad (25)
\end{aligned}
$$

The forward and backward iterations in (26) and (27) are similar to (10) and (11).

$$
\begin{aligned}
\alpha_{t+1}^{kT+T}(j) &= \left[\sum_{i=1}^{N} \alpha_t^{kT+T}(i) \cdot a_{ij} \right] \cdot b_j(o_{t+1}) \qquad (26) \\
\beta_t^{kT+T}(i) &= \sum_{j=1}^{N} a_{ij} \cdot b_j(o_{t+1}) \cdot \beta_{t+1}^{kT+T}(j) \qquad (27)
\end{aligned}
$$

Further developments on the parameter β in (27) is still possible. Replacing the parameters $(k-1)T+T$ and $kT + T$ in (19) for T_1 and T_2 respectively, this equation yields (28). Since the $b_j(o_{kT+T})$ is the same as $P(o_{kT+T} \mid q_{kT+T} = s_j) = \delta_j$, the expression is rewritten as shown in (29), where $\delta_j < 1, \forall j$. From (2), $\sum_{j=1}^{N} a_{ij} = 1$, and so $\sum_{j=1}^{N} a_{ij} \cdot \delta_k = \tau < 1$. For this reason, β is maximized by a decreasing power series where the constant time is τ as stated in (30). This is the clue to choose the appropriate time interval T.

$$
\begin{aligned}
\beta_t^{kT+T}(i) &= \beta_t^{(k-1)T+T}(i) \cdot \beta_{(k-1)T+T}^{kT+T}(i) \\
\beta_t^{kT+T}(i) &= \beta_t^{(k-1)T+T}(i) \sum_{j=1}^{N} a_{ij} b_j(o_{kT+T}) \qquad (28)
\end{aligned}
$$

$$
\begin{aligned}
\beta_t^{kT+T}(i) &\leq \beta_t^{(k-1)T+T}(i) \cdot \sum_{j=1}^{N} a_{ij} \cdot \delta_j \qquad (29) \\
&\leq \beta_t^{(k-1)T+T}(i) \cdot \tau \\
\beta_t^{kT+T}(i) &\leq \tau^k \qquad (30)
\end{aligned}
$$

The necessary tools to evaluate the localization probability were described. To implement this methodology, it is first necessary to define a time interval T. Second, along each time interval the α and β parameters are calculated using the observations and iterative equations. The localization probability for each state, $q_t = s_i, \ i = 1, 2, \cdots, N$ is evaluated using the FB algorithm. The state that maximizes the probability is defined as the current state.

4 Simulation Results

This section presents experimental simulated results on the localization procedure described in Section 3 with the revisited FB algorithm. In the experiments the environment is represented by a topological map with 6 states, $s_1, s_2, \ldots, s_6$, each one characterized by a set of five different attributes, $v_1, v_2, \ldots, v_5$. Two different paths are considered in this map, as represented in Figures 7 and 12. Even though each state is, in general, characterized by a sum of Gaussian pdfs, the present simulation considers, for simplicity, that each state is defined by a single Gaussian pdf. The corresponding mean is a 5-dimensional vector collecting the attribute values and the covariance matrix is diagonal to express a null correlation among the attributes.

To implement the simulation and assess the localization algorithm performance it is necessary to define a referential system that provides the exact position and orientation of the mobile robot along its path and the exact position of each map's state. It should be reinforced, at this stage, that the proposed localization algorithm does not require this metric information, which is exclusively used to simulate the observations, to accomplish the path planning and to evaluate the results. It is considered that the mobile robot is equipped with a sensor whose statistical model is also a one dimensional Gaussian pdf. Again for simulation purposes, it is considered that the probability that the mobile robot observes the attributes of a state is proportional to the inverse of the distance to the state, this distance being evaluated in the metric referential. In addition, it is assumed that the robot's sensor measures, in each time instant, all the attributes of the closest state.

Without a navigation algorithm, which is not the scope of this paper, the performance analysis of the localization procedure requires the *a priori* definition of the path the mobile robot will follow. This path is specified by the set of via points, P_i, illustrated in Figures 7 and 12.

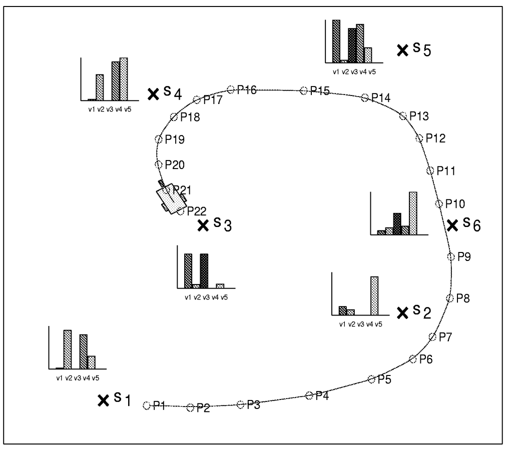

Figure 7: Experiment 1 – Map states, via points, attributes of each state and executed path

As a consequence of the low probability values of localization (caused by the exponential decreasing as referred in (30) in the previous section) the experimental results are displayed in logarithmic scale, $\log P(\cdot)$. At each instant time t, the current state estimate corresponds to the state with the highest probability value. The results displayed in Figures 8-11 and 13 represent the evolution of $\log P(q_t = s_i \mid o_1, \ldots, o_T)$ for $i = 1, 2, \ldots, 6$ along the simulation iterations. The small circles in each function are numbered according to the via points displayed in Figures 7 and 12.

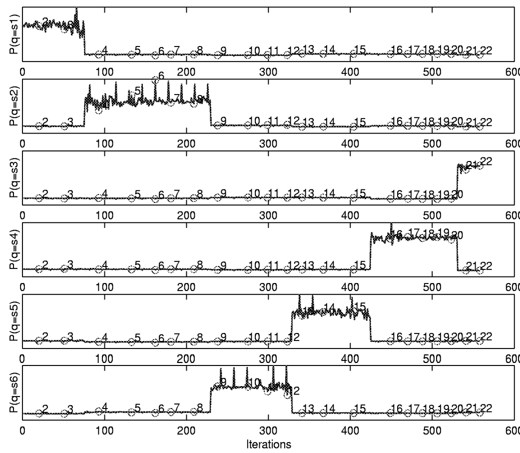

Figure 8: Log of probability localization in each state, with variance noise σ_1^2

Experiment 1

The first experimental results obtained with a observation standard deviation σ_1 are shown in Figure 8. The localization probability is higher when the mobile robot is in the close vicinity of a state. However, when the mobile robot is near to a state, the probability function displays some peak values. These peaks are periodic and dependent on the length of the observation time interval, T, defined in Subsection 3.2. They occur at the beginning of each interval T as consequence of the initialization referred in (24), without any connection with the previous α_{kT}^{kT} value. The smoothing of these peaks is achieved by normalizing the parameter $\alpha_{kT+1}^{kT+T}(i)$ in (24) and replacing it by (31).

$$\alpha_{kT+1}^{kT+T}(i) = \frac{\alpha_{kT}^{kT}(i)}{\sum_{j=1}^{N} \alpha_{kT}^{kT}(j)} \qquad (31)$$

where α_{kT+1}^{kT+T} is equal to the normalization of α parameter of the last iteration on the previous interval. Figure 9 displays the results obtained as in Figure 7 but after the referred normalization. It is clear that the peaky evolution of the probability was decreased, this resulting from the implemented normalization.

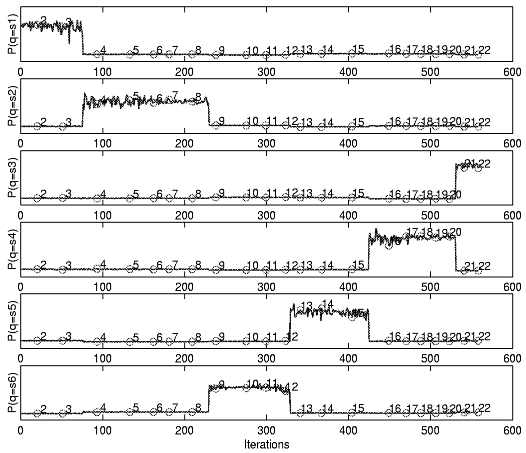

Figure 9: Log of probability localization evolution with normalization in α. Observation variance noise σ_1^2

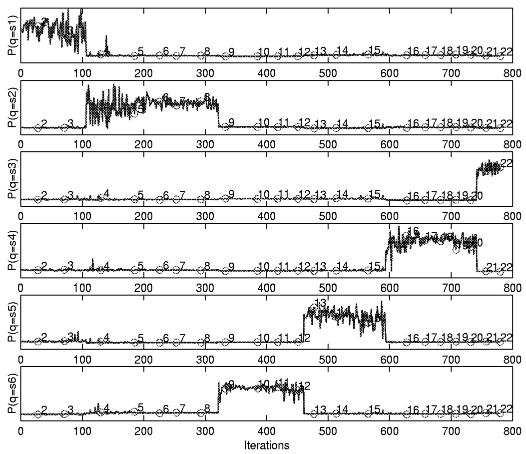

Figure 10: Log of probability localization evolution. Observation variance $\sigma_2^2 = 4\sigma_1^2$

To test the localization algorithm robustness, the standard deviation of the sensor noise was increased. The results in Figure 10 were obtained with the α normalization and a observation standard deviation $\sigma_2 = 2\sigma_1$, where σ_1 refers to the value used in Figures 8 and 9. The results in Figure 11 consider $\sigma_3 = 5\sigma_1$. It is clear the performance degradation in localization estimate as the observation noise is higher. In particular, in Figure 11, the localization estimate is not *stable* when the mobile robot travels between the states s_1 and s_2.

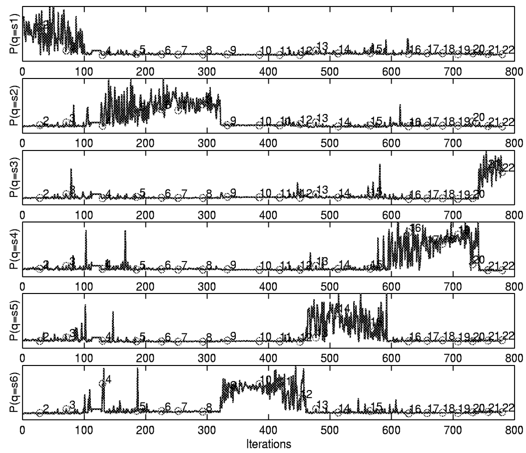

Figure 11: Log of probability localization evolution. Observation variance $\sigma_3^2 = 25\sigma_1^2$

Experiment 2

In Experiment 2 the path planning provides ambiguities. In some segments of the trajectory the mobile robot is equally distant from two or more states as illustrated in Figure 12. For example, the uncertainty situation among the states s_1, s_2 and s_3 is evident between the via points P_3 and P_4, as illustrated in Figure 13 (between the iterations 100 and 200). Also between P_4 and P_6 the uncertainty between s_2 and s_3 remains (iterations 200 to 300). After crossing the via point P_{12} the mobile robot becomes equidistant to the states s_3 and s_4 this creating, due to sensor noise, an uncertainty situation evident in Figure 13 (iterations 800 to 950).

5 Future Development

In this paper, a localization algorithm for a mobile robot was presented aiming at outdoor operation. The algorithm integrates a topological map and a Markov Model in a localization approach. Using the same map representation and a probabilistic approach, this work will proceed with the navigation

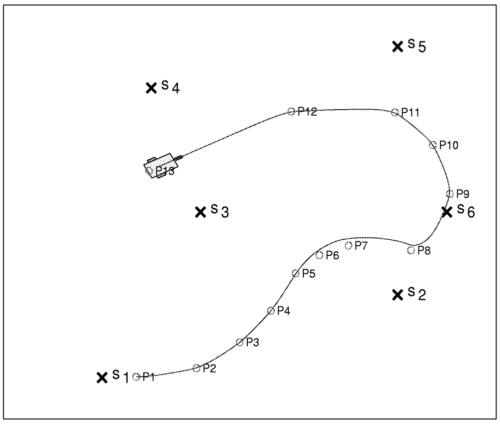

Figure 12: Experiment 2 – Map states, via points and executed path

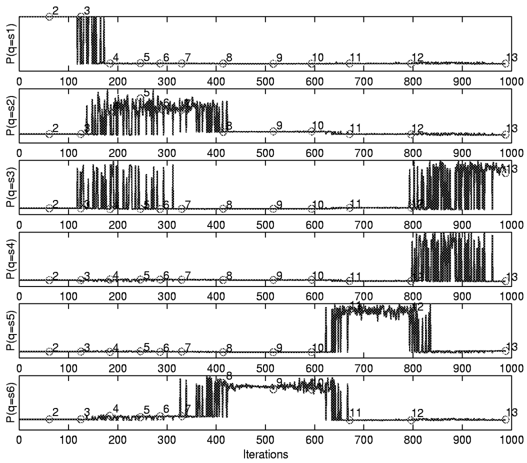

Figure 13: Experiment 2 - Probability localization evolution. The variance noise is σ_2^2

issues, as well as experimental results in a real environment, using a mobile robot in an outdoor scenario. The target application is a rescue mission in an hostile environment.

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